Generalized second law of thermodynamics in warped DGP braneworld

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We investigate the validity of the generalized second law of thermodynamics on the $(n-1)$-dimensional brane embedded in the $(n+1)$-dimensional bulk. We examine the evolution of the apparent horizon entropy extracted through relation between gravitational equation and the first law of thermodynamics together with the matter field entropy inside the apparent horizon. We find that the apparent horizon entropy extracted through connection between gravity and the first law of thermodynamics satisfies the generalized second law of thermodynamics. This result holds regardless of whether there is the intrinsic curvature term on the brane or a cosmological constant in the bulk. The observed satisfaction of the generalized second law provides further support on the thermodynamical interpretation of gravity based on the profound connection between gravity and thermodynamics.

I. INTRODUCTION

Inspired by the profound connection between the black hole physics and thermodynamics, there has been some deep thinking on the relation between gravity and thermodynamics in general for a long time. The pioneer work was done by Jacobson who showed that the gravitational Einstein equation can be derived from the relation between the horizon area and entropy, together with the Clausius relation $\delta Q = T\delta S$ [1]. Further studies on the connection between gravity and thermodynamics has been investigated in various gravity theories [2, 3, 4, 5, 6, 7, 8, 9, 10]. In the cosmological context, attempts to disclose the connection between Einstein gravity and thermodynamics were carried out in [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22]. It was shown that the differential form of the Friedmann equation in the FRW universe can be written in the form of the first law of thermodynamics on the apparent horizon. The profound connection provides a thermodynamical interpretation of gravity which makes it interesting to explore the cosmological properties through thermodynamics.

Investigations on the deep connection between gravity and thermodynamics has recently been extended to braneworld scenarios [23, 24, 25]. The motivating idea for disclosing the connection between the thermodynamics and gravity in braneworld is to get deeper understanding on the entropy of the black hole in braneworld. In the braneworld scenarios, gravity on the brane does not obey Einstein theory, thus the usual area formula for the black hole entropy does not hold on the brane. The exact analytic black hole solutions on the brane have not been found so far, so that the relation between the braneworld black hole horizon entropy and its geometry is not known. We expect that the connection between gravity and thermodynamics in the braneworld can shed some lights on understanding these problems. There are two main pictures in the braneworld scenario. In the first picture which we refer as the Randall-Sundrum II model (RS II), a positive tension 3-brane embedded in an 5-dimensional AdS bulk and the cross over between 4D and 5D gravity is set by the AdS radius [26, 27, 28]. In this case, the extra dimension has a finite size. In another picture which is based on the work of Dvali, Gabadadze, Porrati (DGP model)[29, 30], a 3-brane is embedded in a spacetime with an infinite-size extra dimension, with the hope that this picture could shed new light on the standing problem of the cosmological constant as well as on supersymmetry breaking [29, 31]. The recovery of the usual gravitational laws in this picture is obtained by adding to the action of the brane an Einstein-Hilbert term computed with the brane intrinsic curvature. The obtained connection between gravity and thermodynamics in the braneworld shows that the connection is general and not just an accident in Einstein gravity. The correspondence of the gravitational field equation describing the gravity in the bulk to the first law of thermodynamics on the boundary, the apparent horizon also sheds the light on holography, since the Friedmann equation persists the information in the bulk and the first law of thermodynamics on the apparent horizon contains the information on the boundary. The holographic description of braneworld scenarios and the enropy function on the brane have also been explored in [32, 33, 34, 35, 36].

Besides showing the universality of the connection between gravity and thermodynamics by expressing the gravitational field equation into the first law of thermodynamics on the apparent horizon in different spacetimes, it is of

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great interest to examine other thermodynamical principles if the thermodynamical interpretation of gravity from this correspondence is a generic feature. This is especially interesting in the braneworld. In the braneworld, the entropy was extracted through writing the gravitational equation into the first law of thermodynamics on the apparent horizon[24, 25]. Whether this derived entropy satisfies general thermodynamical principles is another interesting way to examine the correctness of the thermodynamical interpretation of the gravity and the validity of the connection between gravity and thermodynamics. In this paper we are going to study the generalized second law of thermodynamics by investigating the evolution of the apparent horizon entropy deduced through the connection between gravity and the first law of thermodynamics together with the matter fields entropy inside the apparent horizon. The generalized second law of thermodynamics is a universal principle governing the universe. Recently the generalized second law of thermodynamics in the accelerating universe enveloped by the apparent horizon has been studied extensively in [37, 38, 39]. For other gravity theories, the generalized second law has also been studied in [40, 41]. In this work we will explore the generalized second law of thermodynamics in braneworld scenarios, regardless of whether there is the intrinsic curvature term on the brane or a cosmological constant in the bulk. If the thermodynamical interpretation of gravity is correct, the deduced apparent horizon entropy from the connection between gravitational equation and the first law of thermodynamics should satisfy the generalized second law.

Our starting point is the n-dimensional homogenous and isotropic FRW universe on the brane with the metric

$$ds^2 = h_{\mu\nu}dx^\mu dx^\nu + \tilde{r}^2 d\Omega_{n-2}^2, \tag{1}$$

where $\tilde{r} = a(t)r$, $x^0 = t, x^1 = r$, the two dimensional metric $h_{\mu\nu}=$diag $(-1, a^2/(1 - kr^2))$ and $d\Omega_{n-2}$ is the metric of $(n - 2)$-dimensional unit sphere. The dynamical apparent horizon which is the marginally trapped surface with vanishing expansion, is determined by the relation $h^{\mu\nu}\partial_\mu\tilde{r}\partial_\nu\tilde{r} = 0$, which implies that the vector $\nabla\tilde{r}$ is null on the apparent horizon surface. The apparent horizon was argued as a causal horizon for a dynamical spacetime and is associated with gravitational entropy and surface gravity [42, 43, 44]. For the FRW universe the apparent horizon radius reads

$$\tilde{r}_A = \frac{1}{\sqrt{H^2 + k/a^2}}. \tag{2}$$

The associated surface gravity on the apparent horizon can be defined as

$$\kappa = \frac{1}{\sqrt{-h}}\partial_a\left(\sqrt{-h}h^{ab}\partial_{ab}\tilde{r}\right), \tag{3}$$

thus one can easily express the surface gravity on the apparent horizon

$$\kappa = -\frac{1}{\tilde{r}_A}\left(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A}\right). \tag{4}$$

The associated temperature on the apparent horizon can be expressed in the form

$$T_h = \frac{|\kappa|}{2\pi} = \frac{1}{2\pi\tilde{r}_A}\left(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A}\right). \tag{5}$$

where $\frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A} < 1$ ensures that the temperature is positive. Recently the Hawking radiation on the apparent horizon has been observed in [45] which gives more solid physical implication of the temperature associated with the apparent horizon.

II. GSL OF THERMODYNAMICS IN RS II BRANEWORLD

Let us start with the Randall-Sundrum (RS II) model in which no intrinsic curvature term on the brane is included in the action. The Friedmann equation for $(n - 1)$-dimensional brane embedded in $(n + 1)$-dimensional bulk in the RS II model can be written [24]

$$H^2 + \frac{k}{a^2} - \frac{2\kappa_{n+1}^2\Lambda_{n+1}}{n(n-1)} - \frac{\mathcal{C}}{a^n} = \frac{\kappa_{n+1}^4}{4(n-1)^2}\rho^2. \tag{6}$$

where

$$\kappa_{n+1}^2 = 8\pi G_{n+1}, \quad \Lambda_{n+1} = -\frac{n(n-1)}{2\kappa_{n+1}^2\ell^2}, \tag{7}$$

Λ_{n+1} is the $(n+1)$-dimensional bulk cosmological constant, $H = \dot{a}/a$ is the Hubble parameter on the brane, and we assume the matter content on the brane is in the form of the perfect fluid in homogenous and isotropic universe,

$$T_{\mu\nu} = (\rho + P)u_\mu u_\nu + P g_{\mu\nu}, \tag{8}$$

where u^μ, ρ and P are the perfect fluid velocity ($u^\mu u_\nu = -1$), energy density and pressure, respectively. Hereafter we assume that the brane cosmological constant is zero (if it does not vanish, one can absorb it in the stress-energy tensor of perfect fluid on the brane). The constant $\mathcal{C}$ comes from the $(n+1)$-dimensional bulk Weyl tensor. Here we are interested in the flat (Minkowskian) and conformally flat (AdS) bulk spacetimes, so that the bulk Weyl tensor vanishes and thus we set $\mathcal{C} = 0$ in the following discussions.

A. Brane embedded in Minkowski bulk

We begin with the simplest case, namely the Minkowski bulk, in which $\Lambda_{n+1} = 0$. We can rewrite the Friedmann equation (6) in the simple form

$$H^2 + \frac{k}{a^2} = \frac{\kappa_{n+1}^4}{4(n-1)^2}\rho^2. \tag{9}$$

In terms of the apparent horizon radius, we can rewrite the Friedmann equation (9) on the brane as

$$\frac{1}{\tilde{r}_A} = \frac{4\pi G_{n+1}}{n-1}\rho, \tag{10}$$

where we have used Eq. (7). Now, differentiating equation (10) with respect to the cosmic time and using the continuity equation

$$\dot{\rho} + (n-1)H(\rho + P) = 0, \tag{11}$$

we get

$$\dot{\tilde{r}}_A = 4\pi G_{n+1}H(\rho + P)\tilde{r}_A^2. \tag{12}$$

One can see from the above equation that $\dot{\tilde{r}}_A > 0$ provided that the dominant energy condition, $\rho + P > 0$, holds. In our previous work [24], we showed that the Friedmann equation can be written in the form of the first law of thermodynamics on the apparent horizon of the brane

$$dE = T_h dS_h + W dV, \tag{13}$$

where $W = (\rho - P)/2$ is the matter work density [42, 43], $E = \rho V$ is the total energy of the matter inside the $(n-1)$-sphere of radius $\tilde{r}_A$ on the brane, where $V = \Omega_{n-1}\tilde{r}_A^{n-1}$ is the volume enveloped by $(n-1)$-dimensional sphere with the area of apparent horizon $A = (n-1)\Omega_{n-1}\tilde{r}_A^{n-2}$ and $\Omega_{n-1} = \frac{\pi^{(n-1)/2}}{\Gamma((n+1)/2)}$. Using this procedure we extracted an expression for the entropy at the apparent horizon on the brane [24]

$$S_h = \frac{2\Omega_{n-1}\tilde{r}_A^{n-1}}{4G_{n+1}}. \tag{14}$$

It is worth noting that the entropy obeys the area formula of horizon in the bulk (the factor 2 comes from the $\mathbb{Z}_2$ symmetry in the bulk). This is due to the fact that because of the absence of the negative cosmological constant in the bulk, no localization of gravity happens on the brane. As a result, the gravity on the brane is still $(n+1)$-dimensional. Let us now turn to find out $T_h\dot{S}_h$:

$$T_h\dot{S}_h = \frac{1}{2\pi\tilde{r}_A}\left(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A}\right)\frac{d}{dt}\left(\frac{2\Omega_{n-1}\tilde{r}_A^{n-1}}{4G_{n+1}}\right). \tag{15}$$

After some simplification and using Eq. (12) we get

$$T_h\dot{S}_h = (n-1)\Omega_{n-1}H(\rho + P)\tilde{r}_A^{n-1}\left(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A}\right). \tag{16}$$

As we argued above the term $\left(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A}\right)$ is positive to ensure $T_h > 0$, however, in the accelerating universe the dominant energy condition may violate, $\rho + P < 0$. This indicates that the second law of thermodynamics $,\dot{S_h} \geq 0$, does not hold. Then the question arises, "will the generalized second law of thermodynamics, $\dot{S_h} + \dot{S_m} \geq 0$, can be satisfied on the brane?" The entropy of matter fields inside the apparent horizon, S_m, can be related to its energy $E = \rho V$ and pressure P in the horizon by the Gibbs equation [46]

$$T_m dS_m = d(\rho V) + PdV = Vd\rho + (\rho + P)dV, \qquad (17)$$

where T_m is the temperature of the energy inside the horizon. We limit ourselves to the assumption that the thermal system bounded by the apparent horizon remains in equilibrium so that the temperature of the system must be uniform and the same as the temperature of its boundary. This requires that the temperature T_m of the energy inside the apparent horizon should be in equilibrium with the temperature T_h associated with the apparent horizon, so we have $T_m = T_h$[46]. This expression holds in the local equilibrium hypothesis. If the temperature of the fluid differs much from that of the horizon, there will be spontaneous heat flow between the horizon and the fluid and the local equilibrium hypothesis will no longer hold. Therefore from the Gibbs equation (17) we can obtain

$$T_h\dot{S_m} = (n-1)\Omega_{n-1}\tilde{r}_A^{n-2}\dot{\tilde{r}}_A(\rho + P) - (n-1)\Omega_{n-1}\tilde{r}_A^{n-1}H(\rho + P). \qquad (18)$$

To check the generalized second law of thermodynamics, we have to examine the evolution of the total entropy $S_h + S_m$. Adding equations (16) and (18), we get

$$T_h(\dot{S_h} + \dot{S_m}) = \frac{1}{2}(n-1)\Omega_{n-1}\tilde{r}_A^{n-2}\dot{\tilde{r}}_A(\rho + P) = \frac{A}{2}(\rho + P)\dot{\tilde{r}}_A. \qquad (19)$$

where $A > 0$ is the area of apparent horizon. Substituting $\dot{\tilde{r}}_A$ from Eq. (12) into (19) we get

$$T_h(\dot{S_h} + \dot{S_m}) = 2\pi G_{n+1}A\tilde{r}_A^2 H(\rho + P)^2. \qquad (20)$$

The right hand side of the above equation cannot be negative throughout the history of the universe. Hence we have $\dot{S_h} + \dot{S_m} \geq 0$ which guarantees that the generalized second law of thermodynamics is fulfilled in a region enclosed by the apparent horizon on the brane embedded in the Minkowski bulk.

B. Brane embedded in AdS bulk

In the previous subsection we assumed that the bulk cosmological constant is absent and hence we saw that no localization of gravity happens on the brane. Let us now leave that assumption by taking $\Lambda_{n+1} < 0$, which is the case of the real RS II braneworld scenario. Using Eq. (7) the Friedmann equation (6) can be written as

$$\sqrt{H^2 + \frac{k}{a^2} + \frac{1}{\ell^2}} = \frac{4\pi G_{n+1}}{n-1}\rho. \qquad (21)$$

In terms of the apparent horizon radius we have

$$\rho = \frac{n-1}{4\pi G_{n+1}}\sqrt{\frac{1}{\tilde{r}_A^2} + \frac{1}{\ell^2}}. \qquad (22)$$

Taking the derivative of the above equation with respect to the cosmic time and using the continuity equation (11), one gets

$$\dot{\tilde{r}}_A = \frac{4\pi}{\ell}G_{n+1}H(\rho + P)\tilde{r}_A^2\sqrt{\tilde{r}_A^2 + \ell^2}. \qquad (23)$$

The entropy expression associated with the apparent horizon in the RS II braneworld with negative bulk cosmological constant can be obtained as [23, 24]

$$S_h = \frac{(n-1)\ell\Omega_{n-1}}{2G_{n+1}}\int_0^{\tilde{r}_A} \frac{\tilde{r}_A^{n-2}}{\sqrt{\tilde{r}_A^2 + \ell^2}}d\tilde{r}_A. \qquad (24)$$

After the integration we have

$$S_h = \frac{2\Omega_{n-1}\tilde{r}_A^{n-1}}{4G_{n+1}} \times {}_2F_1\left(\frac{n-1}{2}, \frac{1}{2}, \frac{n+1}{2}, -\frac{\tilde{r}_A^2}{\ell^2}\right), \tag{25}$$

where ${}_2F_1(a,b,c,z)$ is the hypergeometric function. It is worth noticing when $\tilde{r}_A \ll \ell$, which physically means that the size of the extra dimension is very large if compared with the apparent horizon radius, one recovers the area formula for the entropy on the brane given in Eq. (14). This is an expected result since in this regime we have a quasi-Minkowski bulk and we have shown in the previous subsection that for a RS II brane embedded in the Minkowski bulk, the entropy on the brane follows the $(n+1)$-dimensional area formula in the bulk. Next we turn to calculate $T_h\dot{S}_h$:

$$\begin{aligned} T_h\dot{S}_h &= \frac{1}{2\pi\tilde{r}_A}\left(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A}\right)\frac{d}{dt}\left[\frac{\Omega_{n-1}\tilde{r}_A^{n-1}}{2G_{n+1}} \times {}_2F_1\left(\frac{n-1}{2}, \frac{1}{2}, \frac{n+1}{2}, -\frac{\tilde{r}_A^2}{\ell^2}\right)\right] \\ &= \frac{1}{2\pi\tilde{r}_A}\left(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A}\right)\frac{(n-1)\ell\Omega_{n-1}}{2G_{n+1}}\frac{\tilde{r}_A^{n-2}\dot{\tilde{r}}_A}{\sqrt{\tilde{r}_A^2 + \ell^2}}. \end{aligned} \tag{26}$$

Using Eq. (23), after some simplification we obtain again

$$T_h\dot{S}_h = (n-1)\Omega_{n-1}H(\rho+P)\tilde{r}_A^{n-1}\left(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A}\right). \tag{27}$$

Adding equation (27) with Gibbs equation (18), we reach

$$T_h(\dot{S}_h + \dot{S}_m) = \frac{1}{2}(n-1)\Omega_{n-1}\tilde{r}_A^{n-2}\dot{\tilde{r}}_A(\rho+P) = \frac{A}{2}(\rho+P)\dot{\tilde{r}}_A. \tag{28}$$

Substituting $\dot{\tilde{r}}_A$ from Eq. (23) into (28) we get

$$T_h(\dot{S}_h + \dot{S}_m) = \frac{2\pi}{\ell}G_{n+1}A\tilde{r}_A^2\sqrt{\tilde{r}_A^2 + \ell^2}\ H(\rho+P)^2. \tag{29}$$

The right hand side of the above equation cannot be negative throughout the history of the universe, which means that $\dot{S}_h + \dot{S}_m \geq 0$ always holds. This indicates that the generalized second law of thermodynamics is fulfilled in the RS II braneworld embedded in the AdS bulk.

III. GSL OF THERMODYNAMICS IN DGP BRANEWORLD

In the previous section, we have studied the validity of the generalized second law of thermodynamics in RS II braneworlds embedded in $(n+1)$-dimensional Minkowski and AdS bulks. In this section, we would like to extend the discussion to the DGP braneworld in which the intrinsic curvature term on the brane is included in the action. The generalized Friedmann equation for the DGP model is given by [24]

$$\epsilon\sqrt{H^2 + \frac{k}{a^2} - \frac{2\kappa_{n+1}^2\Lambda_{n+1}}{n(n-1)} - \frac{\mathcal{C}}{a^n}} = -\frac{\kappa_{n+1}^2}{4\kappa_n^2}(n-2)(H^2 + \frac{k}{a^2}) + \frac{\kappa_{n+1}^2}{2(n-1)}\rho, \tag{30}$$

where $\kappa_n^2 = 8\pi G_n$ and $\epsilon = \pm 1$. For later convenience we choose $\epsilon = 1$. Taking the limit $\kappa_n \to \infty$, while keeping κ_{n+1} finite, the equation (30) reduces to the Friedmann equation (6) in the RS II braneworld. Again, we are interested in studying DGP braneworlds embedded in the Minkowski and AdS bulks, and we set $\mathcal{C} = 0$.

A. Brane embedded in Minkowski bulk

In the Minkowski bulk, $\Lambda_{n+1} = 0$, and the Friedmann equation (30) reduces to the form

$$\sqrt{H^2 + \frac{k}{a^2}} = -\frac{\kappa_{n+1}^2}{4\kappa_n^2}(n-2)(H^2 + \frac{k}{a^2}) + \frac{\kappa_{n+1}^2}{2(n-1)}\rho. \tag{31}$$

In terms of the apparent horizon radius, we can rewrite this equation in the form

$$\rho = \frac{(n-1)(n-2)}{2\kappa_n^2}\frac{1}{\tilde{r}_A^2} + \frac{2(n-1)}{\kappa_{n+1}^2}\frac{1}{\tilde{r}_A}. \tag{32}$$

Now, differentiating equation (32) with respect to the cosmic time and using the continuity equation we get

$$\dot{\tilde{r}}_A = 4\pi\tilde{r}_A^2 H(\rho+P)\left(\frac{n-2}{2G_n\tilde{r}_A} + \frac{1}{G_{n+1}}\right)^{-1}. \tag{33}$$

One can see from the above equation that $\dot{\tilde{r}}_A > 0$ provided that the dominant energy condition, $\rho + P > 0$, holds. However this is not always the case in an accelerating universe. The entropy expression associated with the apparent horizon in the pure DGP braneworld can be obtained from the connection between Friedmann equation and the first law of thermodynamics on the apparent horizon [24]

$$S_h = \frac{(n-1)\Omega_{n-1}\tilde{r}_A^{n-2}}{4G_n} + \frac{2\Omega_{n-1}\tilde{r}_A^{n-1}}{4G_{n+1}}. \tag{34}$$

It is interesting to note that in this case the entropy can be regarded as a sum of two area formulas; one (the first term) corresponds to the gravity on the brane and the other (the second term) corresponds to the gravity in the bulk. This indeed reflects the fact that there are two gravity terms in the action of DGP model. Next we turn to calculate $T_h\dot{S}_h$:

$$\begin{aligned} T_h\dot{S}_h &= \frac{1}{2\pi\tilde{r}_A}\left(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A}\right)\frac{d}{dt}\left(\frac{(n-1)\Omega_{n-1}\tilde{r}_A^{n-2}}{4G_n} + \frac{2\Omega_{n-1}\tilde{r}_A^{n-1}}{4G_{n+1}}\right) \\ &= \frac{1}{2\pi\tilde{r}_A}\left(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A}\right)(n-1)\Omega_{n-1}\left(\frac{(n-2)\tilde{r}_A^{n-3}}{4G_n} + \frac{\tilde{r}_A^{n-2}}{2G_{n+1}}\right)\dot{\tilde{r}}_A. \end{aligned} \tag{35}$$

Using Eq. (33), we obtain

$$T_h\dot{S}_h = (n-1)\Omega_{n-1}H(\rho+P)\tilde{r}_A^{n-1}\left(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A}\right). \tag{36}$$

To check the generalized second law of thermodynamics, we have to examine the evolution of the total entropy S_h+S_m. Combining equations (36) with Gibbs equation (18), again we get

$$T_h(\dot{S}_h + \dot{S}_m) = \frac{A}{2}(\rho+P)\dot{\tilde{r}}_A. \tag{37}$$

Substituting $\dot{\tilde{r}}_A$ from Eq. (33) into (37) we get

$$T_h(\dot{S}_h + \dot{S}_m) = 2\pi A\tilde{r}_A^2 H(\rho+P)^2\left(\frac{n-2}{2G_n\tilde{r}_A} + \frac{1}{G_{n+1}}\right)^{-1}. \tag{38}$$

The right hand side of the above equation is always positive throughout the history of the universe. Therefore the generalized second law of thermodynamics $\dot{S}_h+\dot{S}_m \geq 0$ is fulfilled in the DGP braneworld embedded in the Minkowski bulk.

B. Brane embedded in AdS bulk

For the AdS bulk with $\Lambda_{n+1} < 0$, we can write the Friedmann equation (30) in the form

$$\sqrt{H^2 + \frac{k}{a^2} + \frac{1}{\ell^2}} = -\frac{\kappa_{n+1}^2}{4\kappa_n^2}(n-2)(H^2 + \frac{k}{a^2}) + \frac{\kappa_{n+1}^2}{2(n-1)}\rho, \tag{39}$$

where we have used Eq. (7). In terms of the apparent horizon radius, this equation can be rewritten into

$$\rho = \frac{(n-1)(n-2)}{2\kappa_n^2}\frac{1}{\tilde{r}_A^2} + \frac{2(n-1)}{\kappa_{n+1}^2}\sqrt{\frac{1}{\tilde{r}_A^2} + \frac{1}{\ell^2}}. \tag{40}$$

If one takes the derivative of the equation (40) with respect to the cosmic time, after using Eqs. (7) and (11), one gets

$$\dot{\tilde{r}}_A = 4\pi \tilde{r}_A^2 H(\rho + P) \left(\frac{n-2}{2G_n \tilde{r}_A} + \frac{\ell}{G_{n+1}\sqrt{\tilde{r}_A^2 + \ell^2}} \right)^{-1}. \tag{41}$$

When the dominant energy condition holds, $\dot{\tilde{r}}_A > 0$. The entropy expression associated with the apparent horizon of the DGP brane embedded in the AdS bulk can be extracted through relating the Friedmann equation to the first law of thermodynamics on the apparent horizon [24]

$$S_h = (n-1)\Omega_{n-1} \int_0^{\tilde{r}_A} \left(\frac{(n-2)\tilde{r}_A^{n-3}}{4G_n} + \frac{\ell}{2G_{n+1}} \frac{\tilde{r}_A^{n-2}}{\sqrt{\tilde{r}_A^2 + \ell^2}} \right) d\tilde{r}_A. \tag{42}$$

After integration it reads

$$S_h = \frac{(n-1)\Omega_{n-1}\tilde{r}_A^{n-2}}{4G_n} + \frac{2\Omega_{n-1}\tilde{r}_A^{n-1}}{4G_{n+1}} \times {}_2F_1\left(\frac{n-1}{2}, \frac{1}{2}, \frac{n+1}{2}, -\frac{\tilde{r}_A^2}{\ell^2} \right). \tag{43}$$

Again it is interesting to see that in the warped DGP brane model embedded in the AdS bulk, the entropy associated with the apparent horizon on the brane has two parts. The first part follows the n-dimensional area law on the brane and the second part is the same as the entropy expression obtained in RS II model. The calculation of $T_h\dot{S}_h$ yields

$$T_h\dot{S}_h = \frac{1}{2\pi\tilde{r}_A} \left(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A} \right) (n-1)\Omega_{n-1} \left[\frac{(n-2)\tilde{r}_A^{n-3}}{4G_n} + \frac{\ell}{2G_{n+1}} \frac{\tilde{r}_A^{n-2}}{\sqrt{\tilde{r}_A^2 + \ell^2}} \right] \dot{\tilde{r}}_A. \tag{44}$$

Substituting $\dot{\tilde{r}}_A$ from Eq. (41), we obtain

$$T_h\dot{S}_h = (n-1)\Omega_{n-1}H(\rho + P)\tilde{r}_A^{n-1} \left(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A} \right). \tag{45}$$

Adding equation (45) to (18), one gets

$$T_h(\dot{S}_h + \dot{S}_m) = \frac{A}{2}(\rho + P)\dot{\tilde{r}}_A. \tag{46}$$

Inserting $\dot{\tilde{r}}_A$ from Eq. (41) into (46) we reach

$$T_h(\dot{S}_h + \dot{S}_m) = 2\pi A \tilde{r}_A^2 H(\rho + P)^2 \left[\frac{n-2}{2G_n \tilde{r}_A} + \frac{\ell}{G_{n+1}\sqrt{\tilde{r}_A^2 + \ell^2}} \right]^{-1}, \tag{47}$$

which cannot be negative throughout the history of the universe and hence the general second law of thermodynamics, $\dot{S}_h + \dot{S}_m \geq 0$, is always protected on the DGP brane embedded in the AdS bulk.

IV. SUMMARY AND DISCUSSIONS

To conclude, we have investigated the validity of the generalized second law of thermodynamics for the $(n-1)$-dimensional brane embedded in the $(n+1)$-dimensional bulk. In the braneworld, the apparent horizon entropy was extracted through the relation between the Friedmann equation to the first law of thermodynamics [24]. We have examined the time evolution of the derived apparent horizon entropy together with the entropy of matter fields enclosed inside the apparent horizon on the brane. We have shown that the extracted apparent horizon entropy through the connection between Friedmann equation and the first law of thermodynamics satisfies the generalized second law of thermodynamics, regardless of whether there is the intrinsic curvature term on the brane or a cosmological constant in the bulk. The validity of the generalized second law of thermodynamics on the brane further supports the thermodynamical interpretation of gravity and provides more confidence on the profound connection between gravity and thermodynamics.

Finally, we must mention that as one can see from Eqs. (16), (27), (36) and (45), the variation of the horizon entropy takes the same form in different braneworlds. This fact sheds the light on holography. The details of the shape of the Hubble parameter (or the Friedmann equation) differ in different braneworld models; this is the bulk effect. However, when they project on the boundary, on the horizon entropy, these differences are simplified, which just hid in the H, $\tilde{r}_A$ etc, while the information on the boundary (the horizon entropy) evolves in the same form. This is similar to the topological black holes, different topology will not change the entropy form, $S = A/4$, on the black hole horizon.

Acknowledgments

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